Scalar meson spectroscopy: achievements and traps

Robert Kamiński, Leonard Leśniak* and Benoit Loiseau**

*Henryk Niewodniczański Institute of Nuclear Physics,
PL 31-342 Kraków, Poland
**LPNHE/LPTPE Université P. et M. Curie, 4, Place Jussieu,
75252 Paris CEDEX 05, France

Interactions in three coupled channels: $\pi\pi$, $K\overline{K}$ and $\sigma\sigma$ have been investigated in a wide two-pion effective mass region from the $\pi\pi$ threshold up to 1600 MeV. Analytical structure of amplitudes in all channels has been studied. It was shown that its knowledge is necessary to understand spectrum of scalar mesons and their nature.

§1. Introduction

Scalar mesons play important role in description of low energy hadron physics. Their spectrum and structure is, however, still not clear. In last years the lightest scalar state $f_0(500)$ (or σ) was a subject of intensive studies [1)]. Another state $f_0(980)$ is often regarded as $K\overline{K}$ or multi-quark bound state. Relatively well experimentally known $f_0(1500)$ is a candidate for a glueball.

Interactions between pairs of light mesons are a main source of information about the spectrum and the structure of scalar mesons. In [2)] we have analysed interactions in three coupled channels: $\pi\pi$, $K\overline{K}$ and $\sigma\sigma$ using a separable potential model in two-pion effective mass region from the $\pi\pi$ threshold up to 1600 MeV. The set of 6 solutions was used to study the analytical structure of amplitudes. Spectrum of scalar mesons and their properties (widths, cross sections, branching ratios, coupling constants) were derived.

§2. Model

Solution of a system of Lippmann-Schwinger equations for the meson-meson interaction amplitudes leads to evaluation of Jost's function $D(k_\pi, k_K, k_\sigma)$ which depends on momenta in three coupled channels. In a fully decoupled case (when all interchannel couplings are equal to zero) the Jost function separates into a product of three independent Jost functions

$$D(k_\pi, k_K, k_\sigma) = D(k_\pi)D(k_K)D(k_\sigma) \qquad (2\cdot1)$$

and for example the $\pi\pi$ element of the S-matrix reads

$$S_{\pi\pi} = \frac{D(-k_\pi)}{D(k_\pi)} = e^{2i\delta_\pi}, \qquad (2\cdot2)$$

where δ_π is the $\pi\pi$ phase shift. Due to general relation

$$D(k_\pi, k_K, k_\sigma) = D^*(-k_\pi^*, -k_K^*, -k_\sigma^*) \qquad (2\cdot3)$$

and relation (2·2) zeroes of the numerator and the denominator ($S_{\pi\pi}$ zeroes and poles, respectively) lie symmetrically with respect to the real axis in the complex momenta planes. Hereafter they will be called original poles and zeroes. Cancellation of moduli of the numerator and the denominator leads to inelasticity $\eta = |S_{\pi\pi}| = 1$ what corresponds to elastic scattering.

In a coupled case (when some interchannel couplings are different from zero) the diagonal S-matrix elements can be expressed as ratios of two Jost functions. For example the $S_{\pi\pi}$ element has a form:

$$S_{\pi\pi} = \frac{D(-k_\pi, k_K, k_\sigma)}{D(k_\pi, k_K, k_\sigma)}. \tag{2·4}$$

Presence of the interchannel couplings leads to a movement of the S-matrix zeroes and poles from their original positions to other positions in the coupled case. Interchannel couplings lead also to splitting of an original S-matrix zero or pole into 2^{n-1} poles and zeroes (n is a number of coupled channels). The S-matrix singularities lie on sheets denoted by signs of imaginary parts of complex momenta ($Imk_\pi Imk_K Imk_\sigma$). Some of those poles and zeroes in the coupled case can lie close enough to physical region to have a significant influence on phase shifts and inelasticities. Such singularities correspond to resonances with parameters related to the pole positions k_p by:

$$4(Rek_p + iImk_p)^2 + 4m_\pi^2 = M^2 - iM\Gamma, \tag{2·5}$$

where M is mass of a resonance and Γ is its width.

§3. Results

Knowledge of connections between positions of the S-matrix singularities in the fully coupled case and their original positions is necessary to investigate a spectrum and structure of scalar mesons. In our model we study positions of singularities as function of the interchannel couplings which we gradually decrease starting from their values in the fully coupled case. All values of the interchannel couplings are determined by our fits to the $\pi\pi$ and $K\overline{K}$ data (see [2]). In Fig. 1 one can see traces of two S-matrix poles for one of our solutions. When all the interchannel couplings go to zero both poles change sheets on their ways to their original position. One of them (number XIV) lies reasonably close to imaginary axis in the complex kaon momentum plane and may be treated as the $K\overline{K}$ quasibound state. In Fig. 1 its original pole lies, however, below the real axis what is typical for an ordinary resonance. It was pointed out in [2] that a lack of precise experimental data near the $K\overline{K}$ threshold leads to two possibilities for the $f_0(980)$ state: solutions with and without the $K\overline{K}$ bound state which have similar values of χ^2 about 1 for one degree of freedom.

In Fig. 2 we show a comparison of data with another solution up to 1600 MeV. Above 1600 MeV one sees a flat behaviour of $\delta_{\pi\pi}$. This can be explained by the fact that one S-matrix pole (number XIII in Table 5 in [2]) and the $S_{\pi\pi}$ zero (corresponding

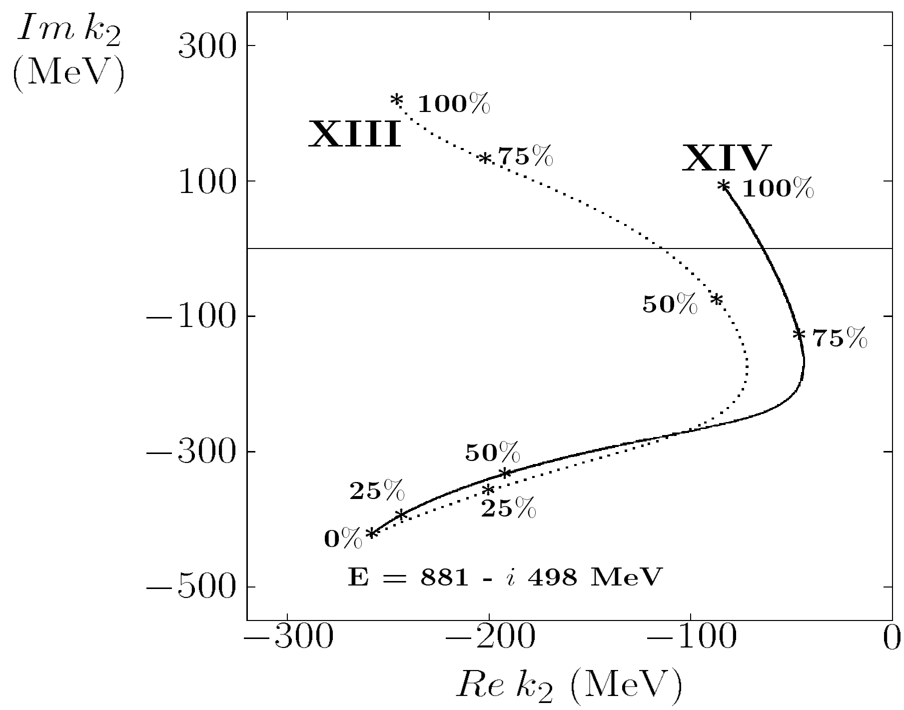

Fig. 1. Example of pole trajectories in the complex kaon momentum plane as a function of the percentage of the interchannel coupling strength for solution A in[2]. Roman numbers correspond to poles in Table 3 therein.

to pole number XVI there) lie on the same sheet at $m_{\pi\pi} = ReE \approx 1660$ MeV and reasonably near by the physical region. Their close positions lead to an effective cancellation of their influence on the $\pi\pi$ phase shifts and explain why there is no apparent indication of any resonance in the theoretical curve in the region above 1600 MeV (see Fig. 2).

Finding of poles and zeros which have the most significant influence on phase shifts and inelasticities is important in identification of scalar resonances. All the S-matrix elements have the same denominator (see for example eq. (2·4) for $S_{\pi\pi}$) so the S-matrix poles are common for all channels. Zeroes of numerators have, however, different positions which depend on a channel. In Table I we present averaged masses and widths of resonances found in our analyses (see [2]). They have been calculated for the S-matrix poles lying in proximity of the physical region. Relatively small errors indicate a stability of the pole positions in all our solutions. It can be seen in Table I that two important poles appear near the $\sigma\sigma$ threshold. Although they lie on

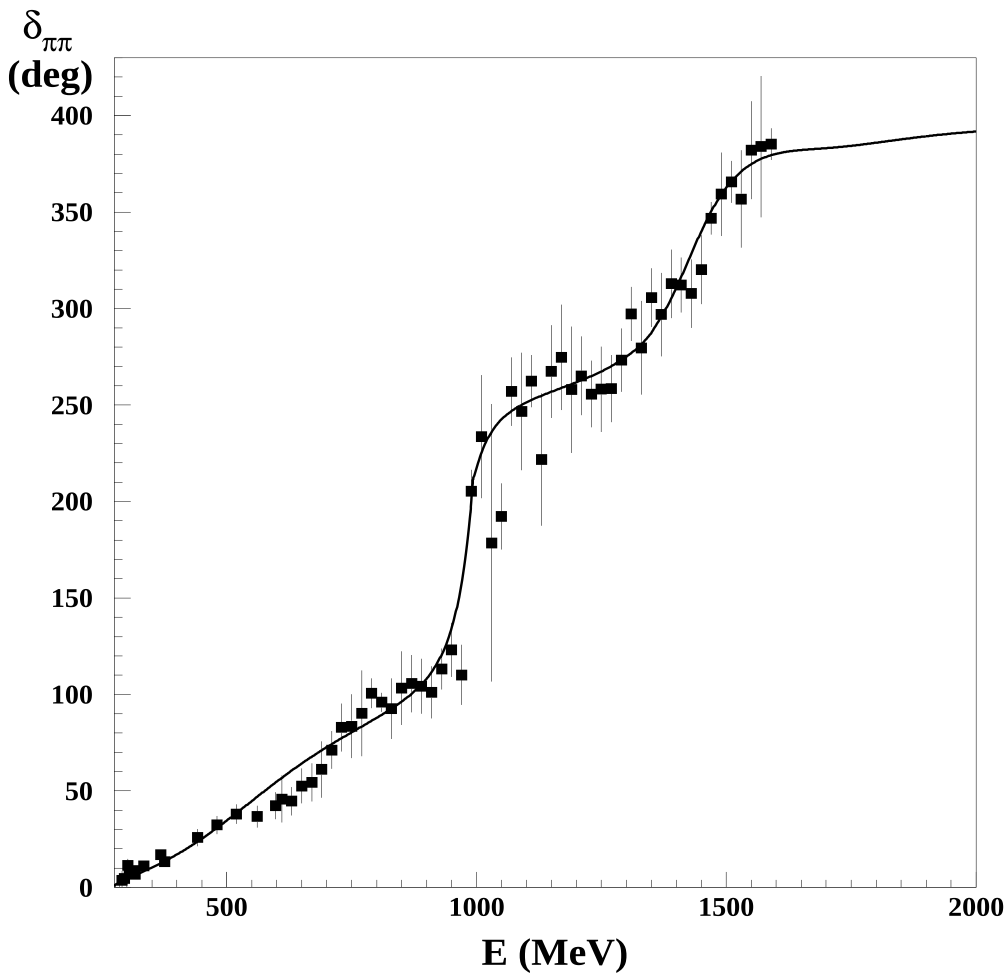

Fig. 2. Energy dependence of $\pi\pi$ phase shifts for solution E

Table I. Average masses and widths of resonances $f_0(500)$, $f_0(980)$ and $f_0(1400)$ found in our solutions A, B, E and F from [2]. Errors represent the maximum departure from the average.

resonance	mass (MeV)	width (MeV)	sheet
$f_0(500)$ or σ	523 ± 12	518 ± 14	$-++$
$f_0(980)$	991 ± 3	71 ± 14	$-++$
	1406 ± 19	160 ± 12	$---$
$f_0(1400)$	1447 ± 27	108 ± 46	$--+$

different sheets both should be taken into account in determination of the $f_0(1400)$ resonance parameters.

Knowledge of positions of all the S-matrix poles and zeroes allows us to describe behaviour of phase shifts (see analyses [2] and [3]). In Fig. 3 one can see contributions

corresponding to particular resonances associated with single poles and a part related to the double pole at $k = i\beta$ coming from the separable potential term of our two-channel model[3)]. The parameter β is close to 1 GeV and the double pole contribution to $\delta_{\pi\pi}$ is decreasing monotonically down to about -90^o at $k_\pi = \beta$ (at $m_{\pi\pi} \approx 2$ GeV). When the parameter β is small the influence of the potential term on phase shifts in a particular channel is much stronger. As it can be seen in Fig. 4 the double pole at $k_\sigma \approx i93$ MeV produces a very strong decrease of $\delta_{\sigma\sigma}$ phase shifts near the $\sigma\sigma$ threshold for solution E in our three channel model[2)] . It is worthwhile to notice that a similar negative part called "background" has also been proposed by the Ishida group[4)].

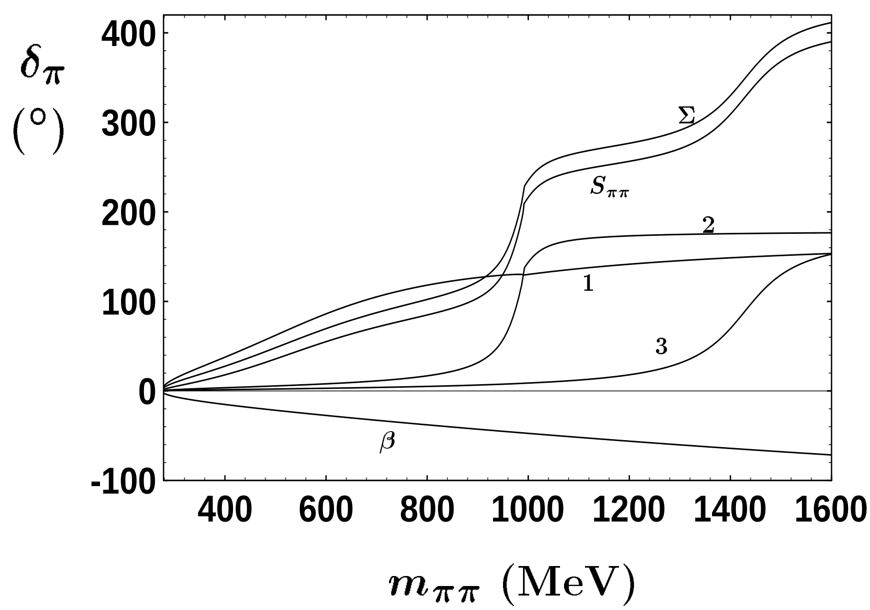

Fig. 3. Effective mass dependence of the $\pi\pi$ phase shifts generated by particular poles and zeroes: 1 - related with $f_0(500)$, 2 - related with $f_0(980)$ and 3 - related with $f_0(1400)$, β denotes part coming from the potential term, Σ - sum of phases from all the poles and $S_{\pi\pi}$ - full dependence of the $\pi\pi$ phase shifts

§4. Conclusions

Analysis of analytical structure of the multichannel amplitudes is necessary to study scalar mesons. Main achievement of that method is finding the S-matrix singularities which have important influence on the phase shifts and inelasticities. The poles can be related to physical resonances. Near the thresholds more than one pole can play an important role in a given channel.

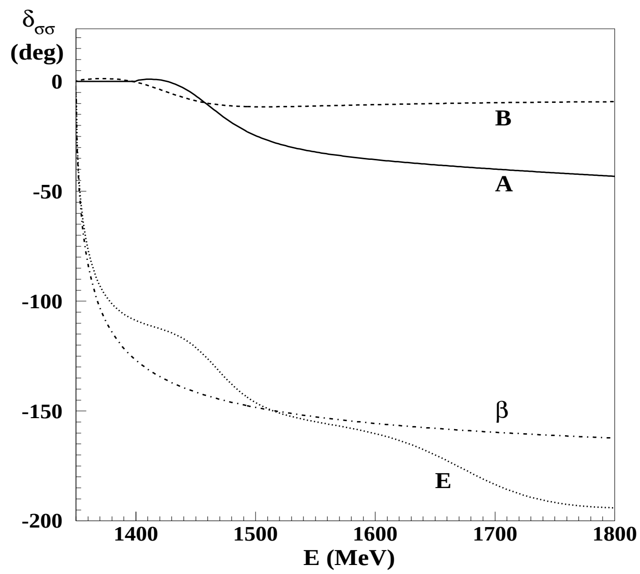

Fig. 4. Effective mass dependence of the $\sigma\sigma$ phase shifts for solutions A, B and E. β denotes the contribution of the potential term for solution E.

Strong interchannel couplings can lead to significant movements of singularities and to a strong mutual cancellation of poles and zeroes lying reasonably close to physical region. One of the main traps in analyses of the multichannel interactions is a difficulty to find unambiguous values of the resonance positions by looking only at the phenomenological energy dependence of phase shifts and inelasticities. Studies of S-matrix singularities are necessary to find parameters of resonances.

In our model total phase shifts can be composed of a sum of parts related to the amplitude poles and zeroes, cuts and the part coming from the potential term. This last part is not arbitrarily added to the scattering amplitudes. It is created automatically by the double S-matrix pole and zero whose positions depend on the range parameters of the separable potentials determined by fits to experimental data.

Acknowledgements

R. Kamiński would like to thank Prof. Shin Ishida and Dr. Muneyuki Ishida for an invitation and hospitality during the workshop.

References

1) Particle Data Group, D. E. Groom *et al.*, Europ. Phys. J. **C 15**, 1 (2000)
2) R. Kamiński, L. Leśniak and B. Loiseau, Europ. Phys. J. **C 9**, 141 (1999)
3) R. Kamiński, L. Leśniak and J. P. Maillet, Phys. Rev. **D50**, 3145 (1994); R. Kamiński, PhD thesis, Institute of Nuclear Physics, Cracow (1996)
4) M. Ishida, Prog. Theor. Phys. 96, 853 (1996)